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FAST TRACK COMMUNICATION

Chiral Swiss rolls show a negative refractive index

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Abstract
Chiral Swiss rolls, consisting of a metal/dielectric laminate tape helically wound on an insulating mandrel, have been developed to form the basis of a highly chiral metamaterial. We have fabricated these elements using a custom-built machine, and have characterized them. We find that the permeability, permittivity, and chirality are all resonant in the region of 80 MHz. The chirality is so strong that it can be directly measured by observing the magnetic response to an applied electric field, and is larger than either the permeability or the permittivity. We have estimated the refractive indices from these data, and find both strong circular dichroism and a wide frequency range where the refractive index is negative.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The engineered response of artificially constructed metamaterials has had a dramatic impact on the physics, optics, and engineering communities, because these metamaterials can offer electromagnetic properties that are difficult or impossible to achieve with conventional, naturally occurring materials. For example, metamaterials based on the 'Swiss roll' [1–3] element have proved to be extremely effective in the radiofrequency (RF) regime. Their low resonance frequency and intense magnetic activity have been exploited to demonstrate RF flux guiding [4, 5] at the resonant frequency. Moreover, their strong response allied with dense packing gives rise to a large negative permeability bandwidth [6], and sub-wavelength imaging [7] has been demonstrated when the permeability $\mu = -1$ [8].

The majority of work on metamaterials has concentrated on the control of their permittivity, ε, and permeability, μ, across a frequency range from the RF through the microwave regime to the optical. When material is made that has both ε and μ negative, the refractive index, n, becomes negative [9], and such materials are sometimes known as 'left-handed'. However, it is important to note that these materials are not chiral; the name [10] refers to the sense of the $\mathbf{E} \times \mathbf{H}$ product with respect to the wavevector $\mathbf{k}$. It is of course possible to make metamaterials that do have a genuine handedness or chirality, and these have in fact been studied for some years. For example, materials based on the incorporation of small wire helices in a non-chiral matrix [11, 12] exhibit chirality in the microwave regime. A further development has been the introduction of planar chiral materials [13]. However in all of these cases, the chirality is still quite weak (albeit significantly stronger than anything available in nature). Nevertheless, they have interesting properties, particularly when the medium is constructed so that ε and μ are approximately zero, and so are less than the chirality, κ, a condition known as chiral nihility [14]. A more interesting material would be one in which the chirality is dominant not because ε and μ are small, but because the chirality itself is large. This material would have massive circular dichroism, and it has been shown [15] that such a strongly chiral metamaterial could offer an alternative route to negative refraction and sub-wavelength imaging. The material can be realized by using the Swiss roll

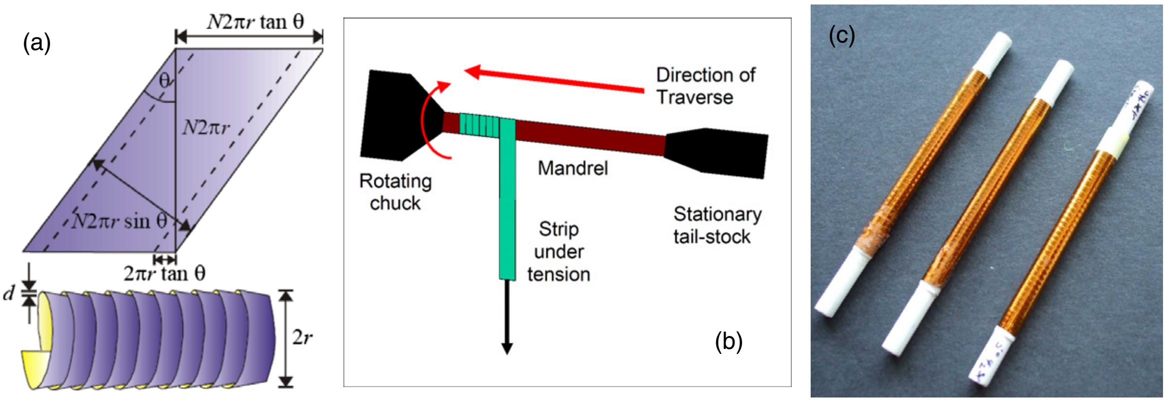

Figure 1. The construction of chiral Swiss rolls. (a) Shows the relations between the mandrel radius, r, the pitch angle, θ, the number of turns, N, and the strip width and overlap. (b) Schematic of the winding process, showing the strip held under tension being wound on an inclined, translating mandrel. (c) A photograph of completed rolls (with the outer wrap removed), showing the helical structure of the elements.

structure, and here we describe an initial investigation of such a medium.

2. Fabrication

In our previous work [4–6, 8] the Swiss rolls were wound and tuned by hand. This is not feasible for the chiral elements: their performance depends critically on the pitch and it is impossible to keep this uniform in a manual process. Therefore a machine was constructed that could wind the rolls in a helical fashion, so that each successive turn extended beyond the underlying layer, thus building up a spiral structure. The mandrel radius, r, the winding angle, θ, and the strip width, $w = N2\pi r \sin\theta$, define the geometry of the element as shown in figure 1(a); here N is the local number of overlapping turns in a cross-section of the element. The pitch of the winding is given by $P = w/N\cos\theta$.

The essential concept of the machine[3] that performs the winding is shown in figure 1(b). A tape of copper–dielectric laminate, under tension from a hanging weight, is wound onto an oblique, rotating mandrel. The strip used here was NovaClad flexible circuit board material (Sheldahl G2202) which consists of 5 μm of copper deposited on 25 μm of polyimide, slit to a 5 mm width. The mandrel traverses at a rate commensurate with the winding angle and the pitch, so that the desired structure is generated. Once the winding is complete, when the roll is ~50 mm long, an adhesive label is wrapped around the structure. The winding angle can be selected in the range $\pm 5°$, and the traverse can be either left-to-right or right-to-left as appropriate, so that we can make either left-handed or right-handed material. The mandrels were glass fibre rods, with diameters of 3, 4 or 5 mm. The process was monitored by measuring the transmission between two loops (not shown); the transmitter loop was placed around the chuck and the receiver loop around the tail-stock. Once the winding had started, a resonant transmission peak was observed, whose frequency fell, initially rapidly and then more slowly, towards the final, stable frequency of the element. A photograph of some completed rolls (without their adhesive labels) is shown in figure 1(c).

We have explored a variety of mandrels sizes and winding angles: smaller mandrels lead to higher frequency and a more limited range of angles. However, for maximum chirality, the winding angle needs to be small [15], so we concentrate here on elements wound at 2° on a 5 mm mandrel; these have a resonant frequency of about 80 MHz.

3. Measurements and results

The definitive signature of a chiral material is the coupling of the electric and magnetic response through the chirality, as seen in the constitutive relations [16]

$$\mathbf{D} = \varepsilon\varepsilon_0\mathbf{E} - \mathrm{i}\sqrt{\varepsilon_0\mu_0}\kappa\mathbf{H}, \qquad \mathbf{B} = \mathrm{i}\sqrt{\varepsilon_0\mu_0}\kappa\mathbf{E} + \mu\mu_0\mathbf{H}. \quad (1)$$

For an isotropic chiral medium, ε, μ and κ are all scalars; for a uniaxial medium having the z-axis unique, ε, μ and κ become diagonal tensors with $\varepsilon_{xx} = \varepsilon_{yy} \neq \varepsilon_{zz}$ and similar relations for μ and κ. Thus the key characteristic is that an exciting magnetic field **H**, or electric field **E**, should induce responses in both **D** and **B** which are both parallel to the exciting field—there are no off-diagonal terms.

Preliminary measurements were made to establish whether the material did indeed show a chirality large enough to measure directly. The rolls were excited by a magnetic field from a loop placed over one end. First, the magnetic response was measured by a loop at the other end. Then an electric dipole antenna was used to explore the induced electric field around the sample. For conventional rolls, this was very small, but for the chiral rolls it was enhanced by 30 dB, with a resonant peak that matched that of the magnetic signal observed with the loop [17]. This clearly demonstrated that these elements had substantial chirality, and so in a second set of experiments, we have measured the permeability, permittivity and chirality of the rolls directly.

First it should be noted that it is not strictly accurate to refer to these properties of a single element: they apply to the bulk. The single element characteristic is the polarizability, which, when averaged over a suitable volume of material

[3] Constructed by 3P Innovation Ltd, Budbrooke Industrial Estate, Warwick, CV34 5WP, UK.

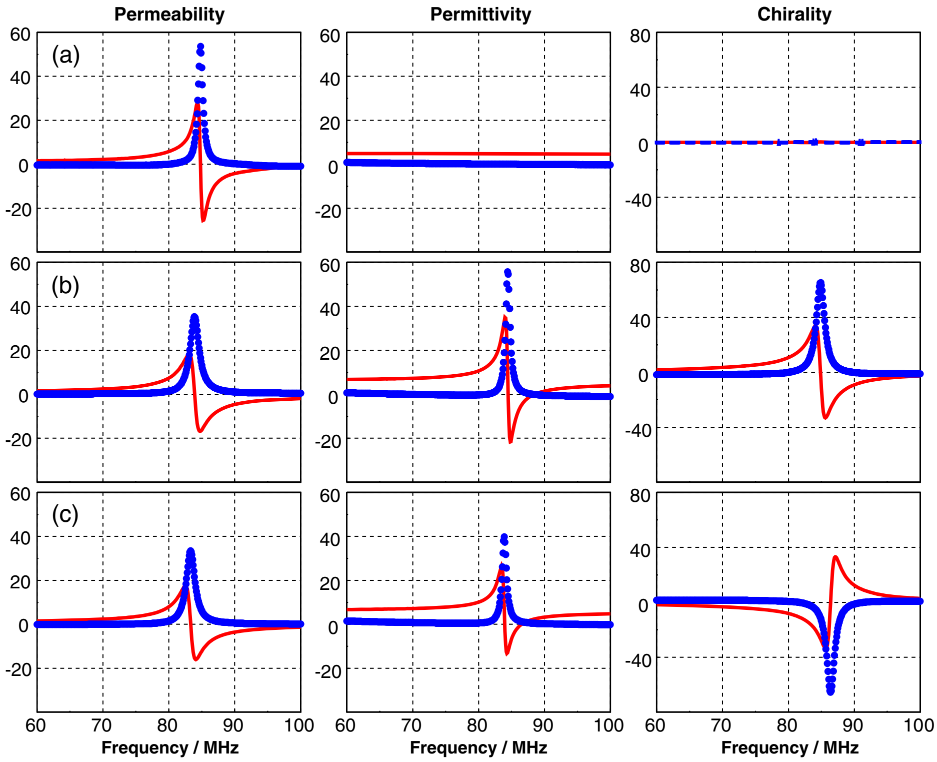

Figure 2. Measured permeability, permittivity and chirality for (a) a conventional, non-chiral roll, (b) a chiral roll, and (c) a chiral roll with the opposite handedness. The full red lines are the real part and the dotted blue lines the imaginary part of the data.

leads to the effective bulk parameters. Nevertheless, it is convenient to refer to the data and its measurement using the bulk nomenclature. Because the sample are rod-shaped, two of these parameters are easier to measure than the third. Referring to (1), we see that a magnetic response **B** can be obtained by applying either a magnetic or an electric field. This response is straightforward to measure, by using a loop around the centre of the rod. Then the signals induced in the loop are simply

$$I = -\mathrm{i}\omega AB = -\mathrm{i}\omega[A_0 + (\mu - 1)A_1]\mu_0 H_0 \quad \text{or} \\ = \omega A_1 \sqrt{\varepsilon_0\mu_0}\kappa E_0 \tag{2}$$

where A_0 is the area of the loop and A_1 the area of the roll, and H_0 and E_0 are the applied magnetic or electric fields, and the response is obtained through μ and κ respectively.

The permeability was found by measuring the signal induced in a loop that was placed in the mid-plane of a pair of Helmholtz coils, which generated a uniform RF magnetic field. Measurements were made as a function of frequency using an Agilent 8753ES network analyzer, both for the empty loop and when a cylindrical Swiss roll sample was inserted in the loop. The permeability was derived from the ratio of the 'sample in' to 'sample out' signals, according to (2), and corrected for effect of the finite length of the sample.

To measure the chirality, the loop was placed mid-way between two large (200 mm square) parallel plates, which generated a uniform electric field away from their edges. The plates were separated by 160 mm so that there was no direct coupling between them and the samples, The chirality was derived using (2), subtracting the 'sample out' baseline from the 'sample in' measurement and again correcting for the roll length.

The permittivity was measured by inserting the sample between the two parallel conducting plates, and measuring the change of capacitance. Because the samples are long rods (typically 50 mm long and 6 mm diameter), there are significant corrections due to stray fields and parasitic impedances that have to be compensated in these measurements. The corrections were found by first measuring the capacitance of some known materials (air, water, nylon), and using these to estimate the parasitic impedances and hence correct the sample measurements. Then the required permittivity was obtained by taking the ratio of the results for a bare mandrel and the Swiss roll sample. It was important to check whether the measuring system (i.e. the parallel plate capacitor) coupled to the Swiss roll resonator. When the plates were held well away from the ends of the sample as for the chirality measurement, the resonant frequency in the permittivity matched that observed in the other two measurements, but the signal was too small for quantitative analysis. To increase the measured signal, the plates were pressed on the protruding ends of the mandrels, typically a few mm from the end of the active sample. Unfortunately, the capacitance of the plates was then coupled more strongly to the sample, and reduced its resonant frequency by about 3.5 MHz. We have checked that this is consistent with (unpublished) data on the tuning of conventional 50 and 150 mm long rolls, from which we expect a frequency shift of approximately 4 MHz for the present rolls, in line with the observed value, and the appropriate frequency correction has been applied. Checks were also made to ensure that samples comprising bundles of rods had the same permittivity values as single rods away from any resonance.

Figure 2 shows the results of these measurements. First, we measured a non-chiral roll that shows a resonant response at about 85 MHz only in its permeability; the permittivity is approximately constant across the frequency band of interest, and the chirality is zero (figure 2(a)). The second sample was a chiral roll constructed with a winding angle of 2° on a 5 mm diameter mandrel: this is the lowest frequency element that was made and has the lowest winding angle and hence the largest chirality. There is a resonant response at about 80 MHz in all three parameters (figure 2(b)). The third sample was a roll of the same design as the previous element, but wound with the opposite handedness. Here, the permeability and permittivity are unaffected, but the chirality has been inverted (figure 2(c)).

4. Discussion

The key point to emphasize about these data is the fact that the chirality has been obtained by a direct measurement of the magnetic response to an applied electric field, whereas previously it has had to be deduced from optical rotation measurements. It is so large that it is greater than either the permeability or the permittivity of the material. Moreover, this has been achieved in the RF range rather than the microwave—a significant point because the chirality tends to zero at zero frequency. Chiral metamaterials have previously been constructed using a dispersion of small metallic helices [11, 12], and these provide chirality in the microwave region. For example Ougier *et al* [18] measured a peak chirality of $\kappa = -1.78 + 0.06\mathrm{i}$ at 15 GHz for their sample using an optical rotation technique. Our values are 40 times larger than these at a frequency that is 200 times lower. Varadan *et al* [11] likewise used an optical rotation method, and analysed their data in the Drude–Born–Fedorov (DBF) formalism, in which the constitutive equations become

$$\mathbf{D} = \varepsilon_{\mathrm{DBF}}(\mathbf{E} + \beta\nabla\times\mathbf{E}) \qquad \mathbf{B} = \mu_{\mathrm{DBF}}(\mathbf{H} + \beta\nabla\times\mathbf{H}) \quad (3)$$

and found that $\beta \approx 10^{-4}$ for their material at the resonant frequency of ∼25 GHz. Using the relations derived by Lindell *et al* [19], we calculated the values of β for our data: they are approximately 10^3 larger than those obtained previously.

What impact does this have on the optical properties of the material? We can calculate the refractive index from the measured permittivity, permeability and chirality as

$$n_{\pm} = \sqrt{\varepsilon\mu} \pm \kappa \quad (4)$$

where $n_{\pm}$ are the indices for right and left circularly polarized radiation, respectively. Now, it must be recognized that the measurements are in fact of the polarizabilities, i.e. they assume that the medium fills all space. In practice this is obviously not so, and there is a filling factor [6] that must be inserted in all three of the bulk parameters: for a two-dimensional log-pile of rolls, we expect this to be $F = \pi/8$. The results of using (4) along with the filling factor to calculate the refractive indices are shown in figure 3 for the two different handed samples. First, and most strikingly, there is the wide region with large negative index that is seen for one polarization, which is different in the two cases, thus for the right-handed rolls it is n_+ (figure 3(a)) that has a large negative range above the resonant frequency, whereas for the left-handed roll it is n_- (figure 3(d)). The second point to bring out is that the size of the refractive index is very large: for both chiral rolls the excursion in the real part exceeds 30 (see figures 3(a), (d)). Thirdly, for each sample there is a large negative index for one polarization and a positive index for the other (see figures 3(a), (b) or (c), (d)), and consequently a massive circular birefringence $\Delta n = n_+ - n_- = 2\kappa$ that may be as large as 25, which in terms of optical rotation corresponds to 12.5 turns per wavelength. Thus the chiral Swiss roll system has a large chirality that dominates its optical behaviour, leading to a strongly negative refractive index and massive circular dichroism over a wide range of frequency above the resonance.

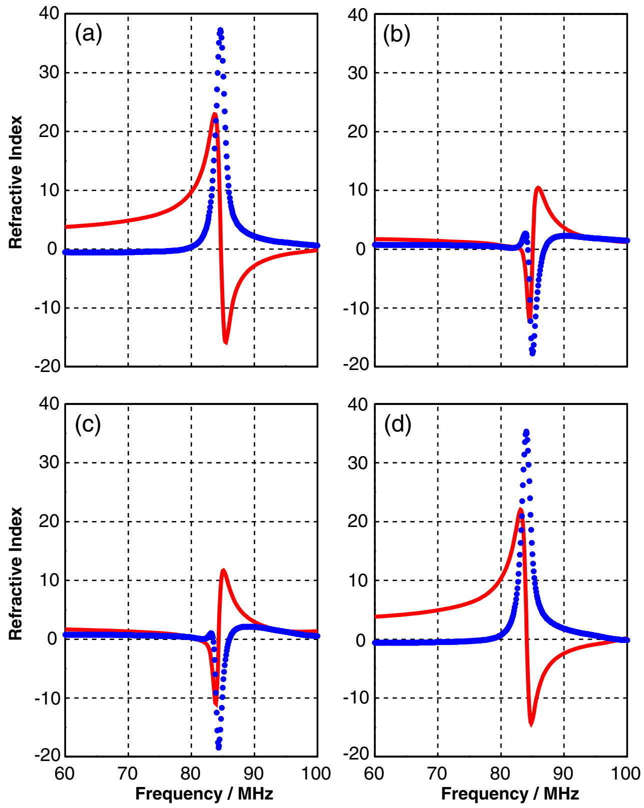

Figure 3. Refractive indices calculated from the measured data of figure 2. (a) and (b) are n_+ and n_- respectively for the right-handed sample shown in figure 2(b) whereas (c) and (d) are those of the left-handed roll of figure 2(c). The full red lines show the real part and the dotted blue lines the imaginary part of the refractive index.

5. Conclusion

In conclusion, we have fabricated a super-chiral medium based on the Swiss roll structure, and have measured its permeability, permittivity and chirality. The chirality is sufficiently strong that it can be measured directly from the magnetic response to an applied electric field, and is larger than either the permeability or the permittivity. These data have been used to calculate the refractive indices of the medium; we find a wide, strongly negative regime above the fundamental resonance, so there is every prospect that a bulk medium constructed from these elements will obey the condition for strong chirality, i.e. $\kappa > \sqrt{\mu\varepsilon}$, and we will be able to demonstrate novel phenomena such as negative refraction [15] in chiral media.

Acknowledgments

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